Chaos Control and Neural Classification

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Chaotic behaviour in biological neural networks is known from various experiments. The recent finding that it is possible to "control" chaotic systems may help answer the question whether chaos plays an active role in neutral information processing. It is demonstrated that a method for chaos control which was proposed by Pyragas can be used to let a chaotic system act like an autoassociative memory for time signal inputs. Specifically a combined chaotic and chaos control system can reconstruct unstable periodic orbits from incomplete information. The potential relevance of these findings for neural information processing is pointed out.

Key words: Chaos control, Neural networks, Classification, Associative memory.

Introduction

The occurrence of chaotic behaviour in neurons and biological neural networks is known from various experiments [1–5]. Such observations raise the question whether chaos is merely a by-product of the complexity of neural behaviour or essential for the function of nervous systems [4, 6]. A possible answer to this question may be derived from the recent findings that it is possible to control chaotic systems [7–14]. The idea of chaos control is to stabilize one out of the infinite number of unstable periodic orbits (UPOs) within a chaotic attractor by means of an appropriate temporal control signal. Since different UPOs can be stabilized in this way, controlled chaotic systems can exhibit many different kinds of behaviour. This flexibility could perhaps be used in neural information processing and learning.

Some hints pointing in this direction do already exist in the literature. In 1987, three years before the idea of chaos control became widely known, Skarda and Freeman [4] demonstrated both periodic and chaotic behaviour in the olfactory bulb of rabbits and proposed that periodic states encoded odor information, while the chaotic state served as a "don't know"-state as well as a ground state from which all periodic orbits could be accessed. Conversely, Ott, Grebogi and Yorke (OGY) pointed out in their first paper on chaos control [10] that a chaotic dynamics built into a system may allow for flexibility, and that chaos might therefore be a necessary ingredient of the brain. More recently, Sepulchre and Babloyantz [15], after demonstrating that the OGY method is suitable even for controlling a system of 9 × 9 coupled oscillators, remarked that controlled chaotic systems might be useful as coding devices in cognitive information processing.

In spite of such hopes, until now no suggestions as to *how* the flexibility of chaotic systems could be used in neural information processing by means of chaos control appear to exist in the literature. In the following, some steps toward answering this question will be attempted by showing that a simple chaos control system can act as an auto-associative memory for some temporal sequences.

Choice of Control Mechanism

The aim of using chaos control for neural information processing implies to some constraints on the control mechanism to be used. The optimal mechanism should be flexible and robust and should avoid major analytical and computational efforts. Many authors [16 to 20], including Sepulchre and Babloyantz [15], prefer OGY's method for chaos control. The drawback of the OGY method, in the context of neural networks, is that it needs a rather forbidding amount of information about, and access to, the system to be controlled [21]. Thus, unless a simplified mechanism for imple-

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menting this control algorithm in neural networks can be found, the usefulness of the OGY method in biological systems remains to be established.

Other approaches seem more promising at the moment. In particular, Pyragas [13] has proposed two methods for chaos control based on the idea of a feedback loop. Consider a dynamical system described by a (potentially unknown) set of ordinary differential equations which produces some measurable signals $\boldsymbol{y}(t)$. For the sake of simplicity we restrict attention to one scalar variable $y(t)$ and assume the latter to be available for external forcing by a control signal $F(t)$. Denoting the other variables $\boldsymbol{x}(t)$, the controlled system reads

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_1(y, \boldsymbol{x}), \quad \dot{y} = f_2(y, \boldsymbol{x}) + F(t). \tag{1}$$

In Pyragas' first method (termed "external force control") one needs to know explicitly the y-component, $y_d(t)$, of the desired orbit, $\{\boldsymbol{x}_d(t), y_d(t)\}$. The control signal is the difference between desired and actual system output, $D(t)$, multiplied by a real parameter $K > 0$:

$$F(t) = K[y_d(t) - y(t)] = KD(t). \tag{2}$$

The second method indicated by Pyragas ("delayed feedback control") needs no information on the desired orbit other than its period, τ. In this case, the control signal is the difference between the delayed and the momentary system output:

$$F(t) = K[y(t-\tau) - y(t)] = KD(t). \tag{3}$$

Pyragas showed that it is possible to control several known chaotic systems (e.g. the Rössler [22] and Lorenz [23] system and the forced Duffing oscillator), as well as an experimental system [24], by using these methods. Moreover, both methods are quite insensitive to noise. A certain drawback of Pyragas' methods is that a dependence on initial conditions may arise. This dependence can be reduced by introducing an amplitude limitation in the feedback loop, by setting

$$F(t) = \begin{cases} -F_0, & KD(t) \leq -F_0, \\ KD(t), & -F_0 < KD(t) < F_0, \\ F_0, & KD(t) \geq F_0. \end{cases} \tag{4}$$

The reason behind this restriction is that a large control signal potentially changes the dynamics of the system and may thus distort the desired periodic orbit.

The advantage of Pyragas' approach is that it needs far less information about the chaotic system than do other methods. The external-force control needs only limited (as will be shown below) knowledge of one component of the UPO to be stabilized, while the delayed feedback method is even more modest, needing only the orbit's period. In contrast, OGY's method requires knowledge of the local *dynamics* in a certain Poincaré section around the UPO to be stabilized.

Since both methods by Pyragas need no knowledge of the system dynamics at all, they are simple enough to be expected to be realized in biological systems. They thus offer themselves for use in biological information processing. In the following, a step in this direction will be attempted.

Classification of Temporal Signals via Chaos Control

Chaos control is useful in dealing with temporal sequences that could in principle thought to be represented by the UPOs of a chaotic system. Thus, as a first approach in using chaos control for information processing, it will be shown that Pyragas' methods can be used as an (auto-)associative memory for temporal sequences and for their classification.

To this end it will be assumed that the signal classes to be extracted correspond to some of the UPOs present in a given chaotic system. (This assumption is compatible with the findings of Skarda and Freeman mentioned above.) The results to be presented here were obtained using the Rössler system

$$\begin{aligned} \dot{x} &= -y - z, \\ \dot{y} &= x + 0.2y, \\ \dot{z} &= 0.2 + z(x - 5.7), \end{aligned} \tag{5}$$

whereby the y-variable was used for both output and control.

Pyragas' external force control, in its original form, enables the stabilization of a UPO *when its component* $y_d(t)$ *is known* and fed back into the system. There is already a certain feature of "pattern completion" implicit in this method, for the control signal $y_d(t)$ suffices to restore not only $y(t)$ to its desired shape but also the other variables $\boldsymbol{x}(t)$. This fact, taken alone, would however hardly suffice to make up an information processing device.

In order for chaos control to qualify as a classifier for temporal signals, at least two further requirements need to be met. First, there must be some basin of attraction around each stabilized UPO so that every signal from this basin forces the system toward the

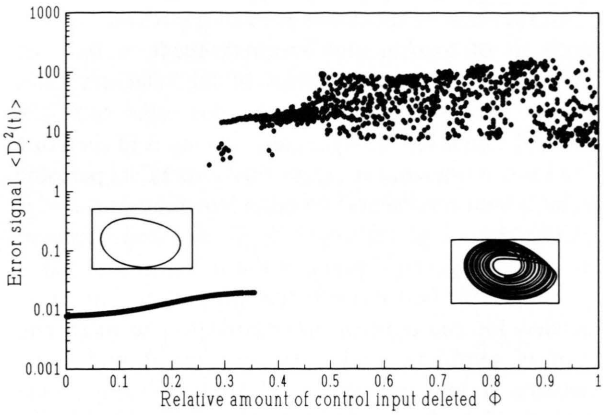

Fig. 1. Dependence of the error signal $\langle D^2(t)\rangle_t$ on the relative amount of information Φ deleted from the control input. Results valid for the period-one orbit of the Rössleer system (5) with external force control, (4), with $K = 0.4$, $F_0 = 0.1$. Integration over 5000 time units, last 200 used for calculating $\langle D^2(t)\rangle_t$. The control orbit starts with the maximum y value. Every dot represents a separate initial condition. The small figures inserted show the resulting orbit of the controlled system for $\Phi = 0.294$ (period-one orbit successfully restored) and $\Phi = 0.992$ (orbit not resto red), respectively. The variation of the error signal in the range [0, 0.3] shows the dependence of the quality of reconstruction on local information of the stored orbit.

UPO, and thus can be recognized as belonging to the corresponding class. Second, control signals far from a basin should not stabilize any UPO in the chaotic system. As a measure of stabilization success, the temporal mean of the squared control signal ("error signal") $\langle D^2(t)\rangle_t$ is suitable.

In order to investigate whether it is possible to stabilize UPOs with *incomplete* information – and to thus perform functions of an associative memory in a chaos control system – the first three UPOs of the Rössler system ($\tau = 5.9$; 11.75; 17.5) were (after prior identification by delayed feedback control) distorted and then used as control inputs. The distortion was accomplished in two different ways: first by deleting a part of the periodic signal used for the control, second by adding noise to the control variable. Further methods have yet to be tested.

In Fig. 1 it is demonstrated that the external-force control is able to restore a UPO even when only a part of the desired orbit $y_d(t)$ is known. First, a UPO $y_d(t)$ of the Rössler system of length τ was obtained by means of delayed feedback control, and stored. This stored orbit was employed for the external force control. Then, to simulate incomplete knowledge about this orbit, the control was periodically switched off during the interval $[2\pi(1-\Phi), 2\pi]$ of the stored orbit. In this way, information was removed from the control, while the period of the control signal was left unchanged.

As can be seen from Fig. 1, the system is quite insensitive to this form of distortion, since the error signal $\langle D^2(t)\rangle_t$ remains small upon deletion of a finite amount of information. The qualitative picture did not change when different parameter settings (like different locations of the part of the orbit presented as control) were used, while the size of the basin of UPO stabilization depended on the orbit used as input as well as on the part of the control signal removed. One expects that these dependencies are correlated with the local Lyapunov exponents along the orbit; this hypothesis has yet to be tested.

In the case of a limited control signal ($F_0 = 0.1$ as in Fig. 1), the transition from stabilization of the desired UPO to identification failure occurs via prolongation of the transients. This leads to the overlap of the two regimes observable in the figure. When the control signal is not limited, transition is more likely to occur by a period doubling bifurcation in the controlled system, leading to the stabilization of a UPO with higher period than the desired one. Further reduction of the input information then sometimes leads to the stabilization of a still higher-periodic orbit, before finally chaotic behaviour is reached. A possible interpretation of this kind of behaviour is that the basins of attraction of the periodic orbits in the controlled system are interlocked. Thus, an input signal outside the basin of the period-one orbit may still suffice to recognize the (next closest) period-two orbit, and so forth.

In either case, since for a given chaotic system the different UPOs $y_i(t)$ can be identified, it is possible to determine the mean squared distance of the orbits i and j, D_{ij}:

$$D_{ij} = \langle [y_i(t) - y_j(t+\delta)]^2 \rangle_{t,\delta} \,. \qquad (6)$$

Using this information one can introduce an acceptance level D_0 for the error signal by setting

$$D_0 = \inf_{i \neq j} D_{ij} \,. \qquad (7)$$

In this way one can rely on the identification of the proper UPO whenever the error signal remains below D_0.

In Fig. 2, incompleteness of knowledge was generated by adding noise to the *control signal*. Specifically, random numbers in the range $[-\frac{2}{5}\mu \ldots \frac{2}{5}\mu]$ have been

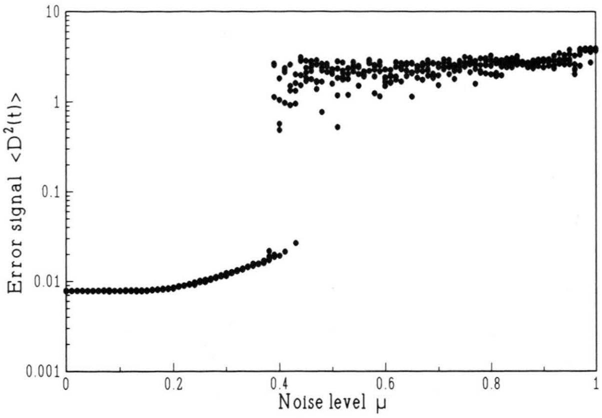

Fig. 2. Dependence of the error signal $\langle D^2(t) \rangle_t$ on the strength of the noise-type distortion μ added to the control signal.

applied at every integration time step. As can be seen from the figure, the effects of the noise-type distortion are similar to those of the information removal shown in Figure 1. A qualitatively similar result was found, when the orbit to be reconstructed was distorted by noise during storage and the distorted stored signal was used as periodic input.

Other authors have already investigated the influence of noise on chaos control. Pyragas, for example, showed that the external-force control method is rather insensitive to noise added to the *dynamic equations* of the controlled system. Similar results have most recently been presented for variants of the OGY method. Hübinger et al. [25] showed that their own control algorithm, which starts out from OGY's method, is suitable to control noisy systems. In the context of this note one could interpret these results as a first hint that classification of temporal signals may be achieveable also with other chaos control methods.

Discussion

It has been found that a method developed by Pyragas [13] for chaos control can be used in the classification of temporal signals. The structure of the example presented is simple enough to permit the prediction that a similar mechanism may be realizable in neural networks and biological systems. It may thus become possible to describe aspects of information processing in neural networks in terms of chaos control.

In the light of the chaos control paradigm, the hypothesis of Skarda and Freeman reads as follows: Without input, the dynamics of the olfactory bulb follows a chaotic attractor. An odor input acts as a control signal on the dynamics, forcing it (if the odor is a known one) into the (previously unstable) periodic orbit which represents the odor information.

The chaotic states found by Skarda and Freeman are an emergent property of the olfactory bulb as a whole [4, 30]. This suggests that the chaotic attractors needed for the control are related to the hierarchy level of neural assemblies. Chaos control might thus become a new possibility of understanding some aspects of the function of natural neural networks on the hierarchy level of cell assemblies. This idea might also be appropriate for the description and analysis of multielectrode recordings [26–29].

A question that needs to be discussed in this context is how learning could be explained in terms of chaos control. One possible approach uses the fact that an infinite number of UPOs is present in every chaotic attractor. Learning could take place by linking a previously unknown input to a new UPO. For establishing such correlations between the external world and control signals, one could make use of Pyragas' *second* (delayed feedback) method. In this way, a coding device for selecting UPOs from a chaotic system using an external signal could be constructed. This can be achieved by modifying a figure of Pyragas (Fig. 1 b in [13]) by introducing many different delay lines instead of just one. Every delay line is activated separately by an external signal. A weakness of this approach is that learning would take place in a separate hardware (dendritic spines?) outside the chaos control system itself.

To conclude, it has been shown that a chaos control system can act as an (auto-)associative memory for some temporal sequences, since it is capable of restoring UPOs present in the chaotic attractor from incomplete or noisy information. In addition to the reconstructed periodic orbit, the resulting error signal $\langle D^2(t) \rangle_t$ can be used as a measure of similarity between input and stored orbit. Because of its capacity to recognize and restore the original UPO from noisy signals, the whole system acts as a classifier. The required degree of similarity between input signal and stored orbit can be adjusted by changing the limitation of the control signal F_0 or the acceptance level D_0. Similar mechanisms may play a role in biological classification and learning.

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